1.5 Describe Angle Pair Relationships

Goal • Use special angle relationships to find angle measures.

Your Notes

VOCABULARY

Complementary angles Two angles whose sum is 90°

Supplementary angles Two angles whose sum is 180°

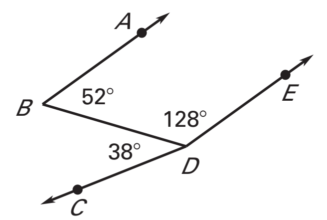
Adjacent angles Two angles that share a common vertex or side, but have no common interior points

Linear pair Two adjacent angles are a linear pair if their noncommon sides are opposite rays.

Vertical angles Two angles are vertical angles if their sides form two pairs of opposite rays.

Example 1 ***Identify complements and supplements***

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because 52° + 38° = 90°, $\angle ABD$ and $\angle CDB$ are complementary angles.

Because 52° + 128° = 180°, $\angle ABD$ and $\angle EDB$ are supplementary angles.

Because $\angle CDB$ and $\angle BDE$ share a common vertex and side, they are adjacent angles.

In Example 1, $\angle BDE$ and $\angle CDE$ share a common vertex. But they share common interior points, so they are *not* adjacent angles.

Your Notes

Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

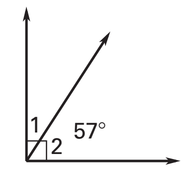
Example 2 ***Find measures of complements and supplements***

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 57°$, find $m\angle 1$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 41°$, find $m\angle 3$.

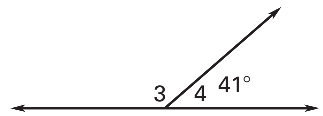
Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.



$m\angle 1 = \underline{90°} - \underline{m\angle 2} = \underline{90°} - \underline{57°} = \underline{33°}$

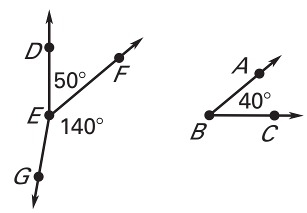
b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.



$m\angle 3 = \underline{180°} - \underline{m\angle 4} = \underline{180°} - \underline{41°} = \underline{139°}$

Checkpoint **Complete the following exercises.**

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



complementary: $\angle DEF$ and $\angle ABC$;
supplementary; $\angle FEG$ and $\angle ABC$;
adjacent: $\angle DEF$ and $\angle FEG$

2. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 73°$, find $m\angle 2$.

17°

3. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 37°$, find $m\angle 3$.

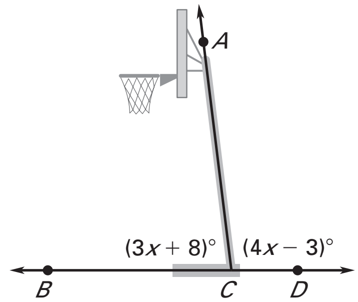
143°

Your Notes

In a diagram, you can assume that a line that looks straight *is* straight. In Example 3, B, C, and D lie on $\overleftrightarrow{BD}$. So, $\angle BCD$ is a straight angle.

Example 3 *Find angle measures*

Basketball The basketball pole forms a pair of supplementary angles with the ground. Find $m\angle BCA$ and $m\angle DCA$.



Solution

Step 1 **Use** the fact that 180° is the sum of the measures of supplementary angles.

$m\angle BCA + m\angle DCA = 180°$	**Write equation.**
$(3x + 8)° + (4x - 3)° = 180°$	**Substitute.**
$7x + 5 = 180$	**Combine like terms.**
$7x = 175$	**Subtract.**
$x = 25$	**Divide.**

Step 2 **Evaluate** the original expressions when $x = 25$.

$m\angle BCA = (3x + 8)° = (3 \cdot 25 + 8)° = 83°$.

$m\angle DCA = (4x - 3)° = (4 \cdot 25 - 3)° = 97°$.

The angle measures are 83° and 97°.

Checkpoint Complete the following exercise.

4. In Example 3, suppose the angle measures are $(5x + 1)°$ and $(6x + 3)°$. Find $m\angle BCA$ and $m\angle DCA$.

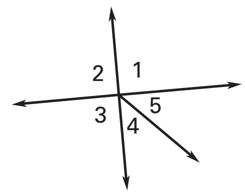
81° and 99°

Your Notes

In the diagram, one side of $\angle 1$ and one side of $\angle 4$ are opposite rays. But the angles are not a linear pair because they are not adjacent.

Example 4 ***Identify angle pairs***

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

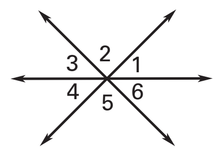
$\angle 1$ and $\angle 3$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

$\angle 1$ and $\angle 2$ are a linear pair. $\angle 2$ and $\angle 3$ are a linear pair.

Checkpoint **Complete the following exercise.**

5. Identify all of the linear pairs and all of the vertical angles in the figure.

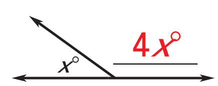


linear pairs: none; vertical angles: $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$

You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

Example 5 ***Find angle measures in a linear pair***

Two angles form a linear pair. The measure of one angle is 4 times the measure of the other. Find the measure of each angle.



Solution

Let $x°$ be the measure of one angle. The measure of the other angle is $4x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

$x° + 4x° = 180°$ **Write an equation.**

$5x = 180$ **Combine like terms.**

$x = 36$ **Divide each side by 5.**

The measures of the angles are $36°$ and $4(36°) = 144°$.

Your Notes

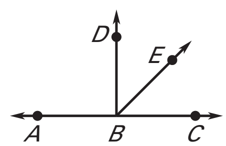
✔ ***Checkpoint*** **Complete the following exercise.**

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the meaure of each angle.

$45°$ and $135°$

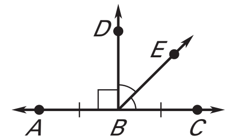
CONCEPT SUMMARY: INTERPRETING A DIAGRAM

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right.



- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point B.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

In the diagram above, you cannot conclude that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



Homework